Student's name **Student's number** **Teacher's name**

________________________ ___________ _____________________

PLC PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2015
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen Black is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 100

Section I: Pages 3-6
10 marks

- Attempt questions 1-10, using the answer sheet on page 23.
- Allow about 15 minutes for this section

Section II: Pages 7-19
90 marks

- Attempt questions 11-16, using the Answer Booklets provided.
- Allow about 2 hours 45 minutes for this section.

Multiple Choice	11	12	13	14	15	16	Total
							%

BLANK PAGE

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

---

1. $\left(\dfrac{2a}{3b}\right)^{-5} = \ ?$

(A) $\dfrac{2a^5}{3b^5}$

(B) $\dfrac{3b^5}{2a^5}$

(C) $\dfrac{243b^5}{32a^5}$

(D) $\dfrac{1}{243b^5}$

2. Let α and β be the solutions of $2x^2 - 5x - 9 = 0$. Which value is the answer to $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$?

(A) $-\dfrac{9}{2}$

(B) $-\dfrac{9}{5}$

(C) $-\dfrac{5}{9}$

(D) $\dfrac{5}{2}$

3. Which expression (value) is equal to $\lim_{x \to \infty} \frac{3x^2}{x^3 - x}$?

(A) $\frac{3x}{x^2 - 1}$

(B) 3

(C) $\frac{3}{x-1}$

(D) 0

4. The period and amplitude of $y = 3 \cos 2x$ is:

(A) Amplitude = 2 Period = $\frac{2\pi}{3}$

(B) Amplitude = 3 Period = π

(C) Amplitude = π Period = 3

(D) Amplitude = $\frac{2\pi}{3}$ Period = 2

5. What is the value of x in the equation $\log_a 12 - 2\log_a 2 = \log_a x$?

(A) 6

(B) $\frac{1}{3}$

(C) 3

(D) $\frac{1}{6}$

6. Which expression shows $\cos^2\left(\frac{\pi}{2}-\theta\right)\cot\theta$ simplified fully ?

(A) $\cos^2\theta\cot\theta$

(B) $\sin\theta\cos\theta$

(C) $\dfrac{\sin^3\theta}{\cos\theta}$

(D) $\sin^2\theta\cot\theta$

7. Which expression is equal to $\int_2^7 \frac{5}{x}\,dx$?

(A) $5(\log_e 7-\log_e 2)$

(B) $\frac{1}{5}(\log_e 7-\log_e 2)$

(C) $\frac{5}{49}-\frac{5}{4}$

(D) 0

8. Which expression is the equation of the normal to the curve $x^2 = 4y$ at the point where $x = 2$?

(A) $y = 1$

(B) $x - y - 1 = 0$

(C) $y = -1$

(D) $x + y - 3 = 0$

9. The function of $g(x)$ is given by

$$g(x)=\begin{cases} x^2-4 & \text{for } x>0 \\ (X) & \text{for } (Y) \end{cases}$$

Which expressions for (X) and (Y) are correct, if $g(x)$ is an odd function?

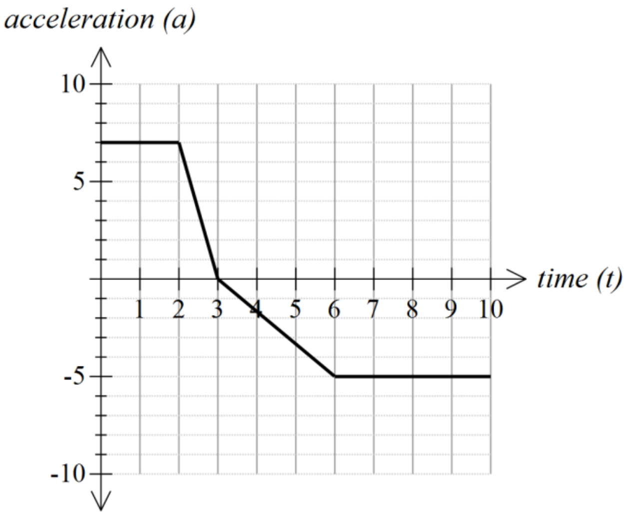
(A) (X): $4-x^2$, (Y): $x<0$

(B) (X): $-x^2-4$, (Y): $x<0$

(C) (X): $4-x^2$, (Y): $x>0$

(D) (X): $-x^2-4$, (Y): $x>0$

10. A particle moves along a straight line. Initially it is at rest at the origin. The graph shows the acceleration, a, of the particle as a function of time t seconds for $0 \le t \le 10$.



At what time during the interval $0 \le t \le 10$ is the particle furthest from the origin?

(A) 3 seconds

(B) 6 seconds

(C) 7 seconds

(D) 8 seconds

End of Section I

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a **<u>new writing booklet</u>**. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a new writing booklet for Question 11.

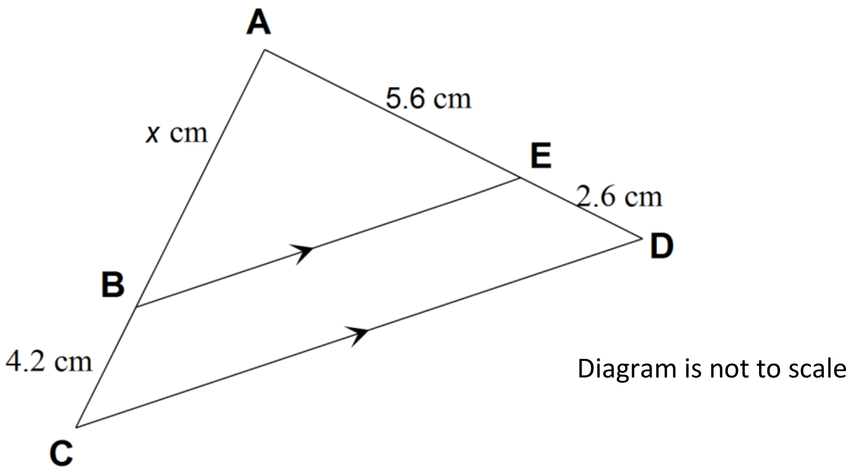
(a) Solve $x^2 - 2x - 7 = 0$, expressing your answer in simplest surd form. **2**

(b) Find $\int \frac{3x}{x^2 + 1}\, dx$. **1**

(c) Simplify fully : **2**

$$\frac{2}{\sqrt{7} + 3} - \frac{3\sqrt{7}}{\sqrt{7} - 3}$$

(d) Find the value of x (correct to the nearest mm). **2**



(e) Find the coordinates of the vertex and focus of the parabola $x^2 - 5y + 5 = 0$. **2**

Question 11 continues on page 8

Question 11 (continued)

(f) Water flows into an empty container, so that after t minutes the volume V of water in litres is given by **3**

$$V = \frac{12t^2}{t+4} \quad \text{for } t \geq 0.$$

What is the rate at which the water is flowing into the container at 1 minute?

(g) Evaluate $\int_0^{\ln 6} e^x \, dx$. **2**

(h) Differentiate $y = \sin 4x$ **1**

End of Question 11

Question 12 (15 marks) Use a new writing booklet for Question 12.

(a) Differentiate:

(i) $y = x^3 e^{3x}$. **2**

(ii) $y = \dfrac{e^x}{(x+3)^2}$. (Full simplification of your answer is not required.) **2**

(b) Solve $\sqrt{3}\cos x = \sin x$ for $0 \le x \le 2\pi$. **2**

(c) Use Simpson's Rule with five function values (x is in radians) to find an approximation for **2**

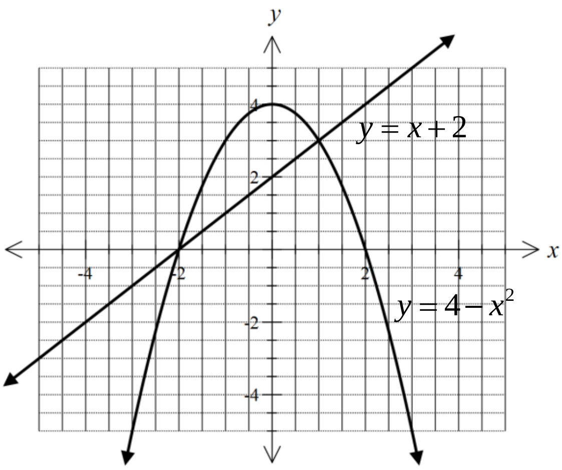
$$\int_0^1 \tan x \, dx.$$

(d) Evaluate **2**

$$\int_0^{\frac{\pi}{2}} \sec^2 3x \, dx$$

Question 12 continues on page 10

(e) Use the graphs below to answer (i) and (ii).



(i) Solve the inequality $4 - x^2 \leq x + 2$. **1**

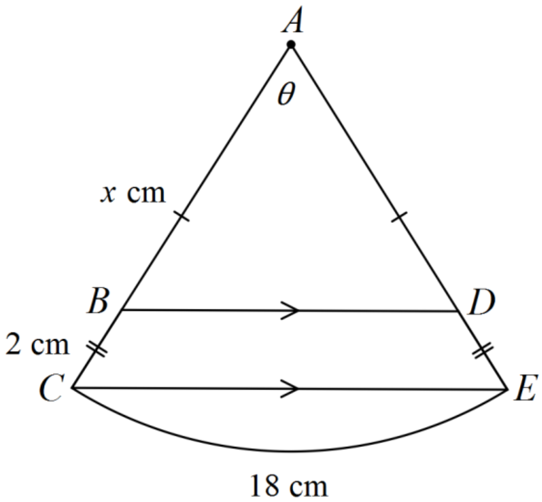
(ii) Calculate the area between the curve $y = 4 - x^2$ and the line $y = x + 2$. **2**

(f) Find the values of A, B and C if $3x^2 + x + 1 \equiv A(x-1)(x+2) + B(x+1) + C$. **2**

End of Question 12

Question 13 (15 marks) Use a new writing booklet for Question 13.

(a) The diagram shows ΔABD and ΔACE, where BD is parallel to CE, $AB = AD = x\,\text{cm}$, $BC = DE = 2\,\text{cm}$ and $AD : AE = 3 : 4$. Triangle ACE and arc CE form a sector in a circle of radius $(x+2)\,\text{cm}$. The angle of the sector is θ radians and arc $CE = 18\,\text{cm}$.

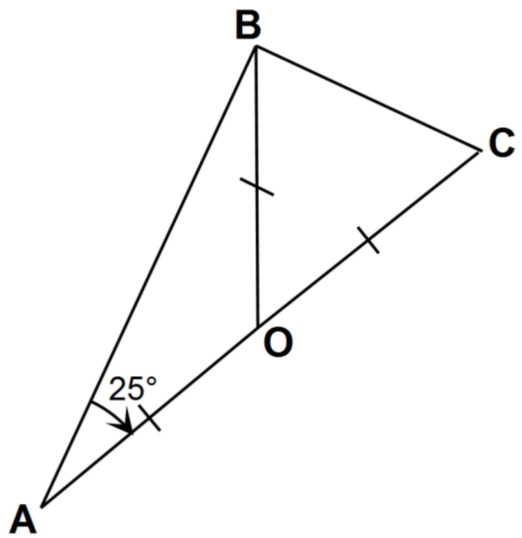


NOT TO SCALE

(i) Find the value of θ. **2**

(ii) Calculate the area of the segment cut off by CE. **2**

(b) In the diagram below, $OA = OB = OC$. Show that $\angle OBC = 65^\circ$. Give reasons. **2**



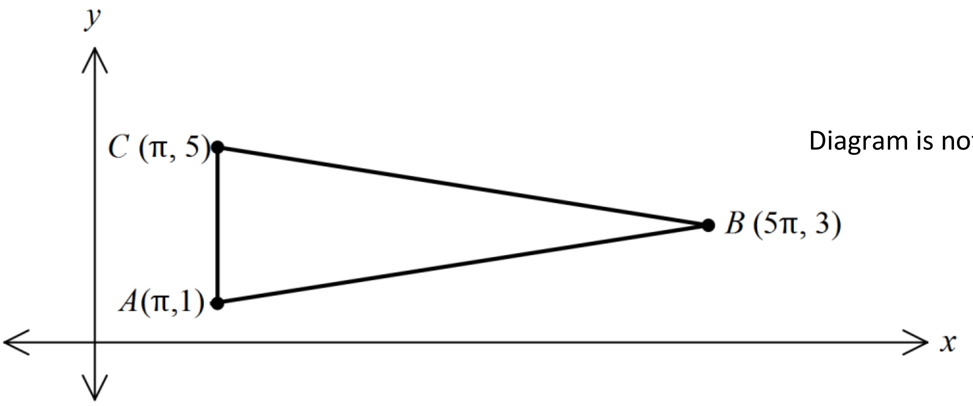
NOT TO SCALE

Question 13 continues on page 12

Question 13 (continued)

(c) For the domain $0 \leq x \leq 6$, a function $y = f(x)$ satisfies $f'(x) < 0$ and $f''(x) < 0$. **2**
Sketch a possible graph of $y = f(x)$ in this domain.

(d) The points $A(\pi, 1)$, $B(5\pi, 3)$ and $C(\pi, 5)$ form an isosceles triangle, with $AB = BC$.



(i) Find the midpoint of AB. **1**

(ii) Show that the equation of the line which is perpendicular to AB and which passes through point C is: **2**

$$y + 2\pi x - 5 - 2\pi^2 = 0$$

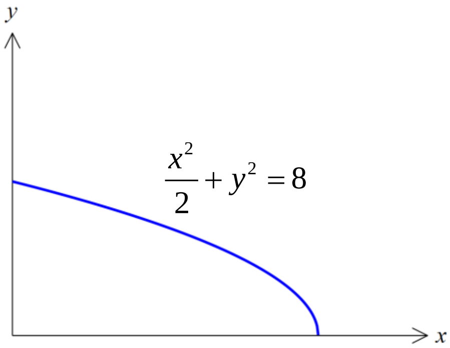
(iii) Calculate the distance AB. **1**

(iv) Using the distances AB, BC and AC, or otherwise, find $\angle CAB$ to the nearest degree. **3**

End of Question 13

Question 14 (15 marks) Use a new writing booklet for Question 14.

(a) The part of the curve $\frac{x^2}{2}+y^2=8$ that lies in the first quadrant is drawn below. **2**



This part of the curve is rotated about the ***x*-axis** to form a solid. Find the exact volume of this solid of revolution.

(b) For the curve $y = x^3(3-x)$

(i) Find all stationary points and determine their nature. **3**

(ii) Draw a sketch of the curve showing the stationary points, inflexion points and intercepts on the axes. **3**

(c) The displacement of a particle moving along the *x*-axis is given by

$$x=5\sin\frac{\pi}{2}t$$

where x is the displacement from the origin in metres, t is the time in minutes and $t \geq 0$.

(i) What is the furthest distance the particle moves away from the origin? **1**

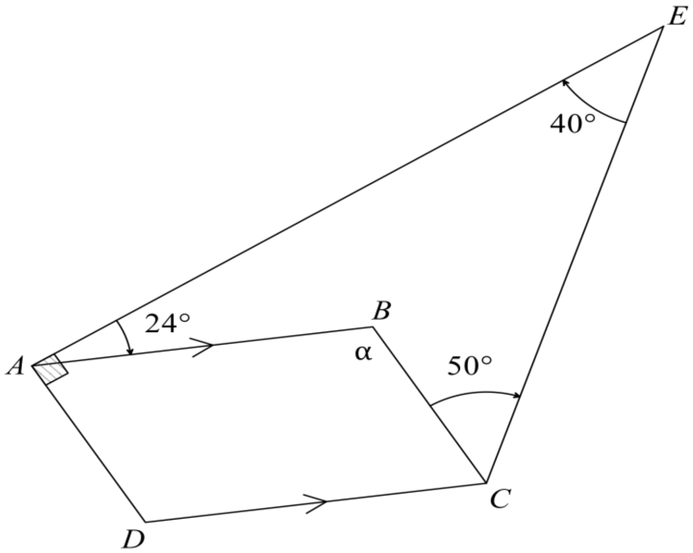
(ii) When does the particle first return to its starting position? **1**

(iii) Find the acceleration of the particle when $t=3\text{min}$. **2**

Question 14 continues on page 14

(d) In the quadrilateral *AECD*, $\angle DAE = 90°$, $\angle AEC = 40°$, $\angle BAE = 24°$ and $\angle BCE = 50°$.

In quadrilateral *ABCD*, *AB* is parallel to *DC* and $\angle ABC = \alpha$ as shown in the diagram.



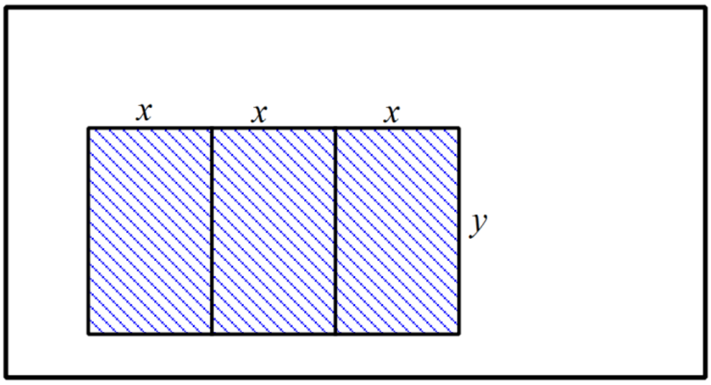
(i) Explain why $\alpha = 114^\circ$. **1**

(ii) Prove that ABCD is a parallelogram. **2**

End of Question 14

Question 15 (15 marks) Use a new writing booklet for Question 15.

(a) Greg has a one hectare block of land (10 000 m^2 = 1 hectare (ha)). He is going to fence off three identical rectangular plots within his block for his three children. Each plot will measure x m by y m as shown in the diagram below. He will retain the remainder of the block for himself and his wife. Greg can only afford 300 m of fencing to go around the children's plots.



(i) Show that $y = 75 - \dfrac{3x}{2}$. **1**

(ii) Find the value of x for which the area of the children's plots will be a maximum. **3**

(iii) Find the maximum area of one of the children's blocks. **1**

(iv) How much of Greg's 1 hectare block is left for him and his wife? **1**

Question 15 continues on page 16

Question 15 (continued)

(b) The acceleration, after t seconds, of a particle moving in a straight line is given by $\ddot{x} = -\dfrac{14}{(t+4)^3}$.

Initially the particle is located $\dfrac{3}{4}$ m to the left of the origin and the initial velocity is $\dfrac{7}{16}$ m/s.

(i) Find the velocity v and the displacement x at any time t. **2**

(ii) What is the velocity of the particle when it passes through the origin? **2**

(iii) Sketch a graph of the displacement as a function of time. **2**

(c) A curve is given by the equation $y = 2x^{\frac{5}{2}} - x^3$, where $x \geq 0$.

(i) Show that $\dfrac{d^2y}{dx^2} = \dfrac{15}{2}\sqrt{x} - 6x$. **1**

(ii) For what value(s) of x is the curve $y = 2x^{\frac{5}{2}} - x^3$ concave up? **2**

End of Question 15

Question 16 (15 marks) Use a new writing booklet for Question 16.

(a) Connor buys a new car, which begins to depreciate immediately. The value ($\$V$) of the car after t years is given by $V = A e^{-kt}$

Where:

A is the initial value

k is the constant of depreciation

t is the time in years

The car is worth $30 000 after 5 years and $18 000 after 10 years.

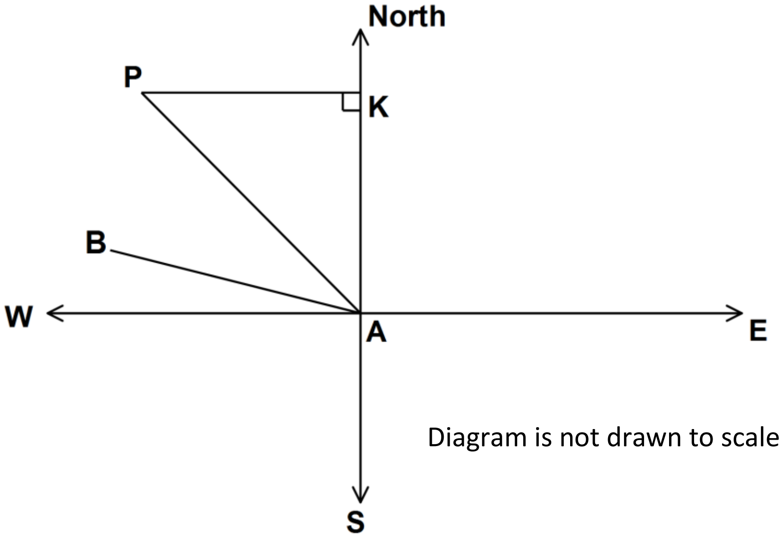
(i) Find the constant of depreciation k. **3**

(ii) Find the initial value of the car. **1**

(iii) How many whole years will it take before the car's value falls below $1 000? **2**

Question 16 continues on page 18

(b) A plane leaves an airport (A) and travels due north $190\sqrt{3}$ kilometres to a point K and then turns due west and travels a further *190* kilometres until it reaches a point P. Due to storms the plane is then diverted to a new airport (B) which is 200 kilometres on a bearing of 280° from A.



(i) Draw the diagram in your answer booklet and label it to show the information.

(ii) Show that $\angle KAP = 30^\circ$. **1**

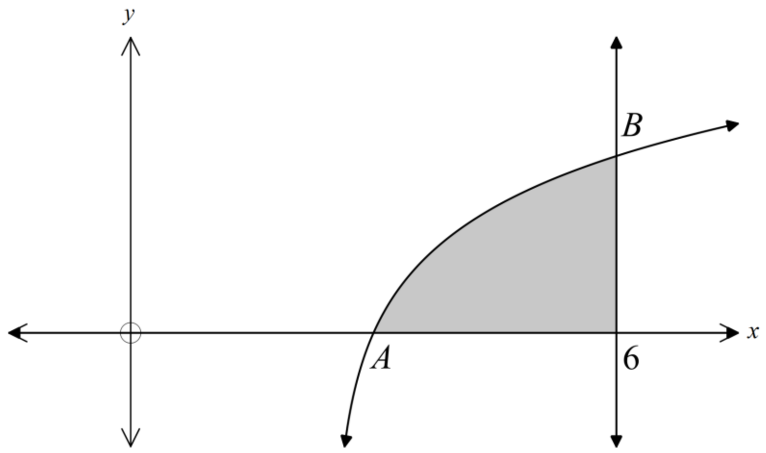
(iii) Show that the plane needs to travel 294 kilometres from P to the new airport (B). **2**

(iv) Hence or otherwise find the bearing (to the nearest degree) on which the plane flies from P to B. **1**

Question 16 continues on page 19

(c) The diagram shows a shaded region which is bounded by the curve $y = \log_e(2x - 5)$, the x axis and the line $x = 6$.

The curve $y = \log_e(2x - 5)$ intersects the x axis at A and the line $x = 6$ at B.



(i) Show that the coordinates of points A and B are (3, 0) and (6, $\log_e 7$) respectively. **1**

(ii) Show that if $y = \log_e(2x - 5)$, then $x = \dfrac{e^y + 5}{2}$. **1**

(iii) Hence find the exact area of the shaded region. **3**

End of Examination

BLANK PAGE

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

BLANK PAGE

Mathematics: Multiple Choice Answer Sheet

Student Number_________

Completely fill the response oval representing the most correct answer.

1.	**A** ○	**B** ○	**C** ○	**D** ○
2.	**A** ○	**B** ○	**C** ○	**D** ○
3.	**A** ○	**B** ○	**C** ○	**D** ○
4.	**A** ○	**B** ○	**C** ○	**D** ○
5.	**A** ○	**B** ○	**C** ○	**D** ○
6.	**A** ○	**B** ○	**C** ○	**D** ○
7.	**A** ○	**B** ○	**C** ○	**D** ○
8.	**A** ○	**B** ○	**C** ○	**D** ○
9.	**A** ○	**B** ○	**C** ○	**D** ○
10.	**A** ○	**B** ○	**C** ○	**D** ○

BLANK PAGE

Academic Year	12 Trial	Calendar Year	2015
Course	Ext 1 Maths	Name of task/exam	Trial HSC

multiple choice

1. $\left(\frac{2a}{3b}\right)^{-5} = \left(\frac{3b}{2a}\right)^{5} = \frac{243b^5}{32a^5}$ (C)

2. $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5/2}{-9/2} = \frac{-5}{9}$ (C)

3. $\lim_{x\to\infty} \frac{3x^2}{x^3-x} = \lim_{x\to\infty} \frac{\frac{3x^2}{x^3}}{\frac{x^3}{x^3}-\frac{x}{x^3}} = \lim_{x\to\infty} \frac{\frac{3}{x}}{1-\frac{1}{x^2}} = 0$ (D)

4. (B)

5. $\log_a 12 - \log_a 2^2 = \log_a\left(\frac{12}{4}\right)$

$= \log_a 3$

$= \log_a x$

$\therefore x = 3$

(C)

6. $\cos^2\left(\frac{\pi}{2}-\theta\right) \times \frac{\cos\theta}{\sin\theta} = \sin^2\theta \times \frac{\cos\theta}{\sin\theta} = \sin\theta\cos\theta$ (B)

7. $\int_2^7 \frac{5}{x}\,dx = 5\ln x\Big]_2^7$

$= 5\ln 7 - 5\ln 2$ (A)

8. $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{x}{2}$

at $x = 2$

$m_T = 1 \quad m_N = -1$ (D)

$y - 1 = -(x - 2)$

$y - 1 = -x + 2$

$x + y - 3 = 0$

9. odd when $f(x) = -f(-x)$

$g(x) = x^2 - 4$

$-g(-x) = -(x^2 - 4)$ (A)

$= -x^2 + 4$

10. (D) Area under graphs above and below axes are equal.

Academic Year	12 Term 1	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

Question 11

a. $x^2 - 2x - 7 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2}$$

$$= \frac{2 \pm \sqrt{32}}{2}$$

$$= \frac{2 \pm 4\sqrt{2}}{2}$$

$$= 1 \pm 2\sqrt{2}$$

b. $\int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int \frac{2x}{x^2+1}\,dx$

$$= \frac{3}{2}\ln(x^2+1) + C$$

c. $\frac{2(\sqrt{7}-3) - 3\sqrt{7}(\sqrt{7}+3)}{7-9} = \frac{2\sqrt{7} - 6 - 21 - 9\sqrt{7}}{-2}$

$$= \frac{-7\sqrt{7} - 27}{-2}$$

$$= \frac{7\sqrt{7} + 27}{2}$$

d. $\frac{x}{4.2} = \frac{5.6}{2.6}$

$$x = \frac{5.6}{2.6} \times 4.2$$

$$x = 9.04\ldots \text{ cm}$$

$$= 90\text{ mm}$$

Academic Year	12 trial	Calendar Year	2015
Course	2unit math	Name of task/exam	Trial HSC

e. $x^2 - 5y + 5 = 0$

$5y = x^2 + 5$

$x^2 = 5y - 5$

$x^2 = 5(y-1)$

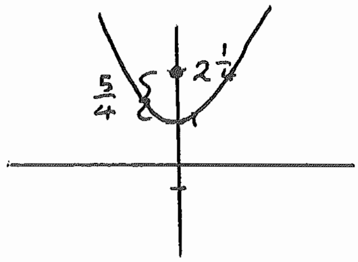
$[x^2 = 4a(y-k)]$

$5 = 4a$

$a = \frac{5}{4}$.

$\therefore$ vertex $(0,1)$

Focus $(0, 2\frac{1}{4})$



f. $V = \dfrac{12t^2}{t+4}$ $t \geq 0$

$u = 12t^2$ $v = t+4$

$u' = 24t$ $v' = 1$

$$\frac{dV}{dt} = \frac{24t(t+4) - 12t^2}{(t+4)^2}$$

$$= \frac{24t^2 + 96t - 12t^2}{(t+4)^2}$$

at $t = 1$

$$\frac{dV}{dt} = \frac{24 + 96 - 12}{5^2}$$

$$= \frac{108}{25} = 4.32 \text{ L/min}$$

g. $$\int_0^{\ln 6} e^x \, dx = e^x \Big]_0^{\ln 6}$$

$= 6 - 1$

$= 5$.

h. $$\frac{dy}{dx} = 4\cos 4x$$

Academic Year	12 Trial	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

Question 12

a (i) $y = x^3 e^{3x}$

$u = x^3 \qquad v = e^{3x}$

$u' = 3x^2 \qquad v' = 3e^{3x}$

$$\frac{dy}{dx} = 3x^2 e^{3x} + 3x^3 e^{3x}$$

$$= 3x^2 e^{3x}(1 + x)$$

(ii) $y = \dfrac{e^x}{(x+3)^2}$

$u = e^x \qquad v = (x+3)^2$

$u' = e^x \qquad v' = 2(x+3)$

$= 2x + 6.$

$$\frac{dy}{dx} = \frac{e^x(x+3)^2 - 2e^x(x+3)}{(x+3)^4}$$

$$= \frac{e^x(x+3)(x+3-2)}{(x+3)^4}$$

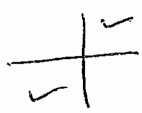
$$= \frac{e^x(x+3)(x+1)}{(x+3)^4}$$

$$= \frac{e^x(x+1)}{(x+3)^3}$$

b. $\sqrt{3}\cos x = \sin x \quad (\div \cos x)$

$\tan x = \sqrt{3}$

$x = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$



c.

	0	1/4	2/4	3/4	1
	0	$\tan\frac{1}{4}$	$\tan 2/4$	$\tan\frac{3}{4}$	$\tan 1$
	y_0	(y_1)	y_2	(y_3)	y_n

$$A = \frac{\frac{1}{4}}{3}\left[0 + \tan 1 + 4(\tan\tfrac{1}{4} + \tan\tfrac{3}{4}) + 2(\tan\tfrac{1}{2})\right]$$

$$= \frac{1}{12}[7.3977\ldots]$$

$$= 0.6164805\ldots$$

$$\doteqdot 0.62 \quad (2dp)$$

Academic Year	12 Trial	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

d. $\int_0^{\frac{\pi}{2}} \sec^2 3x \, dx = \frac{1}{3}\tan 3x \Big]_0^{\frac{\pi}{2}}$

$= \frac{1}{3}\tan\frac{3\pi}{2} - \frac{1}{3}\tan 0$

$= \frac{1}{3}\tan\frac{3\pi}{2}$

$=$ undefined.

e. (i) $x \leq -2$, $x \geq 1$

(ii) $\int_{-2}^{1} 4 - x^2 - (x+2)\, dx = \int_{-2}^{1} 4 - x^2 - x - 2 \, dx$

$= \int_{-2}^{1} 2 - x - x^2 \, dx$

$= 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big]_{-2}^{1}$

$= \left(2(1) - \frac{1}{2} - \frac{1}{3}\right) - \left(2(-2) - \frac{4}{2} - \frac{(-2)^3}{3}\right]$

$= \frac{7}{6} - -\frac{10}{3}$

$= \frac{9}{2}$ units^2

f. $3x^2 + x + 1 \equiv A(x^2 + x - 2) + Bx + B + C$

$\equiv Ax^2 + (A+B)x - 2A + B + C$

$\therefore$ $A = 3$ $\quad A + B = 1$ (1) $\quad -2A + B + C = 1$ (2)

(1) $A + B = 1$
$3 + B = 1$
$B = -2$

(2) $-2A + B + C = 1$
$-2(3) - 2 + C = 1$
$-8 + C = 1$
$C = 9$

$\therefore$ $A = 3 \quad B = -2 \quad C = 9.$

Academic Year	12 Trial	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

Question 13

(i) ① $l = r\theta$

$18 = r \times \theta$

since $x = 6$

$r = 8$

$\therefore 18 = 8 \times \theta$

$\theta = \frac{18}{8} = \frac{9}{4}$

② $\frac{AD}{AE} = \frac{3}{4}$

$\frac{x}{x+2} = \frac{3}{4}$

$4x = 3(x+2)$

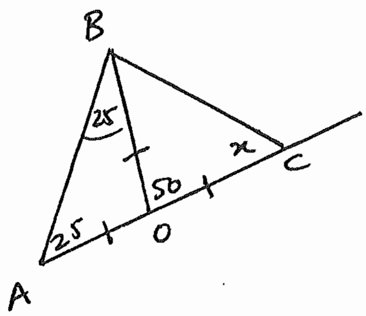
$x = 6$ cm.

(ii) $A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times 8^2 \left(\frac{9}{4} - \sin\frac{9}{4}\right)$

$= 8\left(\frac{9}{4} - \sin\frac{9}{4}\right) \doteq 47.1 \text{ cm}^2$

b.



$OA = OB = OC$ (equal given)

ΔAOB is an isosceles Δ.

$\therefore \angle OAB = \angle OBA = 25°$ (base angles equal)

$\angle BOC = 50°$ (exterior angle of Δ)

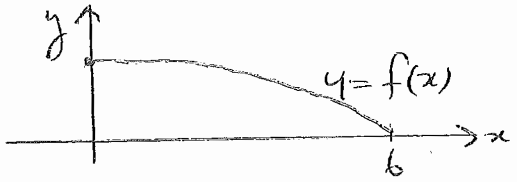
$\angle OBC = \frac{180 - 50}{2}$ (angle sum of Δ) (base angles equal in isosceles Δ)

$\angle OBC = \frac{130}{2}$

$= 65°$.

Academic Year	12 Trial	Calendar Year	2015
Course	2 unit Maths	Name of task/exam	Trial HSC.

c.



d.(i) midpoint AB $= \left(\frac{\pi + 5\pi}{2}, \frac{1+3}{2} \right)$

$= (3\pi, 2)$

A (π, 1)

B (5π, 3)

C (π, 5)

(ii) $m_{AB} = \frac{3-1}{5\pi - \pi} = \frac{2}{4\pi} = \frac{1}{2\pi}$.

⊥ to AB: $m_\perp = -2\pi$.

$m = -2\pi \quad C(\pi, 5)$

$y - 5 = -2\pi(x - \pi)$

$y = -2\pi x + 2\pi^2 + 5$

$y + 2\pi x - 5 - 2\pi^2 = 0$.

(iii) $d_{AB} = \sqrt{(5\pi - \pi)^2 + (3-1)^2}$

$= \sqrt{16\pi^2 + 4}$

$= \sqrt{4(4\pi^2 + 1)}$

$= 2\sqrt{4\pi^2 + 1}$

(iv) Δ ABC is an isosceles Δ

$\therefore AB = BC = 2\sqrt{4\pi^2+1}$, $AC = 4$

$\cos \angle CAB = \frac{4^2 + (2\sqrt{4\pi^2+1})^2 - (2\sqrt{4\pi^2+1})^2}{2 \times 4 \times 2\sqrt{4\pi^2+1}}$

$= \frac{16 + 4(4\pi^2+1) - 4(4\pi^2+1)}{16\sqrt{4\pi^2+1}}$

$= \frac{16}{16\sqrt{4\pi^2+1}}$

$\angle CAB = \cos^{-1}\left(\frac{1}{\sqrt{4\pi^2+1}}\right)$

$\doteqdot 81^\circ$. (nearest degree)

Academic Year	12 Trial	Calendar Year	2015
Course	2 unit Maths	Name of task/exam	Trial HSC

Question 14

a. $\frac{x^2}{2} + y^2 = 8$

$y^2 = 8 - \frac{x^2}{2}$

at $y = 0$.

$0 = 8 - \frac{x^2}{2}$

$\frac{x^2}{2} = 8$

$x = \pm 4.$

$\therefore V = \pi \int y^2 \, dx$

$= \pi \int_0^4 8 - \frac{x^2}{2} \, dx$

$= \pi \left[8x - \frac{x^3}{6} \right]_0^4$

$= \pi \left[(32 - \frac{64}{6}) - (0) \right]$

$= \frac{64\pi}{3}$ units^3

b. $y = x^3(3 - x) = 3x^3 - x^4$

(i) $\frac{dy}{dx} = 9x^2 - 4x^3$

$\frac{d^2y}{dx^2} = 18x - 12x^2.$

$\frac{dy}{dx} = 9x^2 - 4x^3 = 0$ for stat points.

$= x^2(9 - 4x)$

$= 0$

$\therefore x = 0 \qquad 9 - 4x = 0$

$x = \frac{9}{4}$

at $x = 0$ $\quad \frac{d^2y}{dx^2} = 18(0) - 72(0)$

$= 0$

$\therefore$ possible point of inflexion at $x = 0$.

at $x = \frac{9}{4}$ $\quad \frac{d^2y}{dx^2} = 18\left(\frac{9}{4}\right) - 12\left(\frac{9}{4}\right)^2$

$= -\frac{81}{4} < 0 \quad \therefore$ max.

at $x = 0$:

x	-1	0	0
y''	-30	0	6

$\therefore$ point of inflexion (concavity change)

$\therefore$ horizontal point of inflexion

since $\frac{dy}{dx} = 0$ AND $\frac{d^2y}{dx^2} = 0$

at $x = 0$: $\quad y = 0 \quad$ (0, 0) inflex horiz

at $x = \frac{9}{4}$: $\quad y = \left(\frac{9}{4}\right)^3\left(3 - \frac{9}{4}\right)$

$= \frac{2187}{256} \doteqdot 8.543 \quad \left(\frac{9}{4}, 8.543\right)$ MAX.

(ii) at $x = 0$ $\quad y = 0 \quad (0,0)$.

inflexions: $\quad 18x - 12x^2 = 0$

$6x(3 - 2x) = 0$

$x = 0 \quad x = \frac{3}{2}$

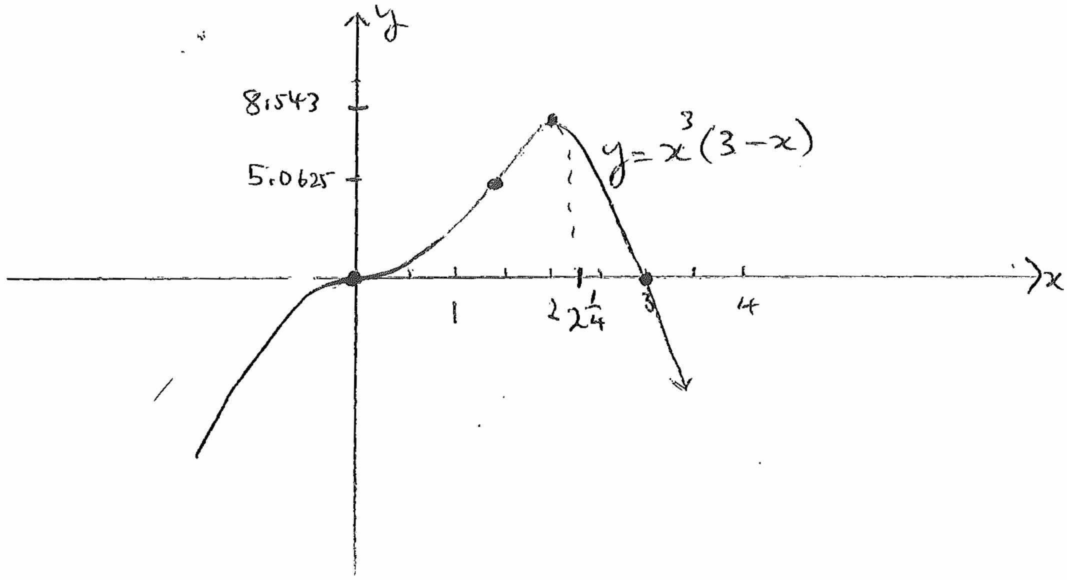
Academic Year	12	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

at $x = \frac{3}{2}$

x	1	1.5	2
y''	6	0	-12
	↑		↓

∴ change in concavity

∴ point of inflexion at $\left(\frac{3}{2}, 5.0625\right)$

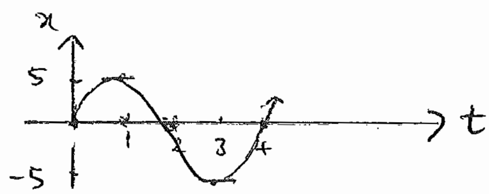


at $y = 0$ $\quad x^3(3-x) = 0$

∴ $x = 0 \quad x = 3 \quad (0,0) \quad (3,0)$

c. $x = 5\sin\frac{\pi}{2}t$

(i) 5 metres



(ii) From graph: after 2 minutes

OR

$x = 0$ ∴ $5\sin\frac{\pi}{2}t = 0$

$\sin\frac{\pi}{2}t = 0$

$\frac{\pi}{2}t = 0, \pi, 2\pi, \ldots$

$t = 0, 2, 4, \ldots$

∴ after 2 minutes.

Academic Year	12	Calendar Year	2015
Course	2unit Trial	Name of task/exam	Trial HSC

(iii) $x = 5\sin\frac{\pi}{2}t$

$\dot{x} = \frac{5\pi}{2}\cos\frac{\pi}{2}t$

$\ddot{x} = -5\left(\frac{\pi}{2}\right)^2\sin\frac{\pi}{2}t$

at $t = 3$ $\quad \ddot{x} = -5\left(\frac{\pi}{2}\right)^2\sin\frac{3\pi}{2}$

$= -5\frac{\pi^2}{4} \times -1$

$= \frac{5\pi^2}{4}$

---

d. (i) reflex $\angle ABC = 360 - (24+50+40) = 246°$

$\alpha = 360 - 246 = 114°$.

(ii) since $\alpha = 114°$

$\angle BCD = 180 - 114$ (cointerior angles are supplementary on parallel lines)

$= 66°$

$\therefore \angle ECD = 66 + 50$

$= 116°$

$\angle ADC = 360 - 116 - 90 - 40$ (angle sum of quadrilateral)

$= 114°$

$\angle BAD = 90 - 24°$ (given)

$= 66°$

$\angle ADC + \angle BCD = 66 + 114$

$= 180°$

$\therefore BC \parallel AD$ (cointerior angles exist)

$\therefore$ Since opposite angles are equal and opposite sides are parallel

Then ABCD is a parallelogram.

Academic Year	12	Calendar Year	2015
Course	2unit Math	Name of task/exam	Trial HSC

Question 15

(i) $6x + 4y = 300$

$4y = 300 - 6x$

$y = \frac{300 - 6x}{4}$

$y = \frac{300}{4} - \frac{6x}{4}$

$y = 75 - \frac{3x}{2}$

(ii) $A = 3xy$

$A = 3x\left(75 - \frac{3x}{2}\right)$

$u = 3x \quad v = 75 - \frac{3x}{2}$

$u' = 3 \quad v' = -\frac{3}{2}$

$\frac{dA}{dx} = 3x\left(-\frac{3}{2}\right) + \left(75 - \frac{3x}{2}\right) \times 3$

$= -\frac{9x}{2} + 225 - \frac{9x}{2}$

$= -\frac{18x}{2} + 225$

$\frac{d^2A}{dx^2} = \frac{-18}{2} < 0 \therefore \text{max.}$

$\frac{dA}{dx} = 0$ for stat. pt.

$-\frac{18x}{2} + 225 = 0$

$x = \frac{225}{9}$

$\therefore$ at $x = \frac{225}{9}$ maximum area

$x = 25$

Academic Year	12	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

∴ maximum area of one of the childrens blocks: $A = xy$

at $x = 25$ $y = 75 - \frac{3}{2}(25)$

$= \frac{75}{2}$

∴ Area $= 25 \times \frac{75}{2}$

$= 937.5 \text{ m}^2$

(iv) Area remaining $= 10000 - 3 \times 937.5$

$= 7187.5 \text{ m}^2$.

Academic Year	12	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

Question 15 continued

b. $\ddot{x} = -\frac{14}{(t+4)^3}$

at $t=0$ $x = -\frac{3}{4}$ $\dot{x} = \frac{7}{16}$ m/s.

(i) $\dot{x} = -14\int \frac{1}{(t+4)^3}\,dt$

$= -14\int (t+4)^{-3}\,dt$

$= -14\left[\frac{(t+4)^{-2}}{-2}\right] + c$

$= 7(t+4)^{-2} + c$

at $t=0$ $\dot{x} = \frac{7}{16}$

$\frac{7}{16} = 7(0+4)^{-2} + c$

$\frac{7}{16} = \frac{7}{16} + c$

$\therefore c = 0$

$\dot{x} = \frac{7}{(t+4)^2}$

$x = 7\int (t+4)^{-2}\,dt$

$x = \frac{7(t+4)^{-1}}{-1} + c$

at $t=0$ $x = -\frac{3}{4}$

Academic Year	12	Calendar Year	2015
Course	2unit maths	Name of task/exam	Trial HSC

$$-\frac{3}{4} = -7(4)^{-1} + c$$

$$c = -\frac{3}{4} + \frac{7}{4}$$

$$c = 1$$

$$\therefore \quad x = -\frac{7}{t+4} + 1$$

(ii) at $x = 0$

$$0 = -\frac{7}{t+4} + 1$$

$$0 = -7 + t + 4$$

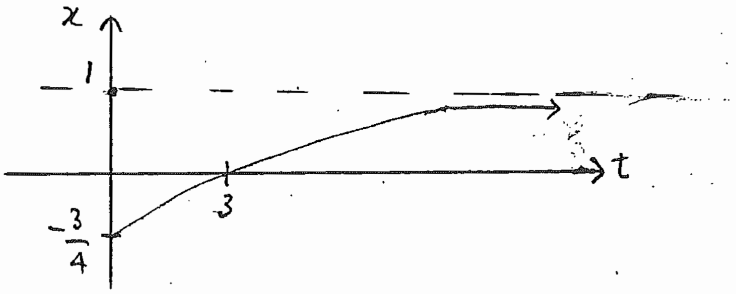
$$\underline{t = 3}$$

$$\therefore \quad \dot{x} = \frac{7}{(t+4)^2}$$

$$= \frac{7}{(7)^2}$$

$$= \frac{1}{7}$$

(iii)



at $t = 0$ $x = -\frac{3}{4}$

$$x = -\frac{7}{t+4} + 1$$

Academic Year	12	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

c. $y = 2x^{\frac{5}{2}} - x^3$

(i) $\frac{dy}{dx} = 5x^{\frac{3}{2}} - 3x^2$

$\frac{d^2y}{dx^2} = \frac{15}{2}x^{\frac{1}{2}} - 6x$

$= \frac{15\sqrt{x}}{2} - 6x$

(ii) $\frac{d^2y}{dx^2} > 0$

$\frac{15}{2}x^{\frac{1}{2}} - 6x > 0$

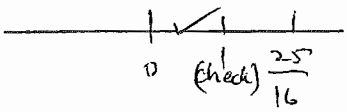
$3x^{\frac{1}{2}}\left(\frac{5}{2} - 2\sqrt{x}\right) > 0$

$-2\sqrt{x} + \frac{5}{2} = 0$

$2\sqrt{x} = \frac{5}{2}$

$4x = \frac{25}{4}$

$x = \frac{25}{16}.$



$\therefore \quad 0 < x < \frac{25}{16}.$

method 2: $\frac{15\sqrt{x}}{2} - 6x > 0$

$\frac{15\sqrt{x}}{2} > 6x$

since $\sqrt{x} > 0$ and $x > 0$ (length)

then $\frac{225x}{4} > 36x^2$ (square both side)

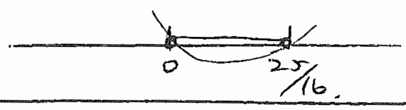
$225x > 144x^2$

$144x^2 - 225x < 0$

$x(144x - 225) < 0$

$144x = 225$

$x = \frac{25}{16}.$



$0 \le x \le \frac{25}{16}.$

Academic Year	12	Calendar Year	2015
Course	2Unit Maths	Name of task/exam	Trial HSC

Question 16.

a. $V = Ae^{-kt}$

$30000 = Ae^{-5k}$

$18000 = Ae^{-10k}$

(i) $\dfrac{Ae^{-5k}}{Ae^{-10k}} = \dfrac{30\,000}{18000}$

$e^{5k} = \dfrac{5}{3}$

$5k = \ln \dfrac{5}{3}$

$k = \dfrac{1}{5} \ln\left(\dfrac{5}{3}\right)$

$\doteqdot 0.102165\ldots$

(ii) $30\,000 = A\, e^{-5\left(\frac{1}{5}\ln\left(\frac{5}{3}\right)\right)}$

$30\,000 = A\, e^{-\ln\left(\frac{5}{3}\right)}$ note: $e^{-\ln\left(\frac{5}{3}\right)} = e^{\ln\frac{3}{5}} = \dfrac{3}{5}.$

$A = \dfrac{30\,000}{\frac{3}{5}}$

$A = \$50\,000$

(iii) $1000 = 50\,000\, e^{-k \times t}$

$\dfrac{1}{50} = e^{-k\times t}$

$-kt = \ln\left(\dfrac{1}{50}\right)$

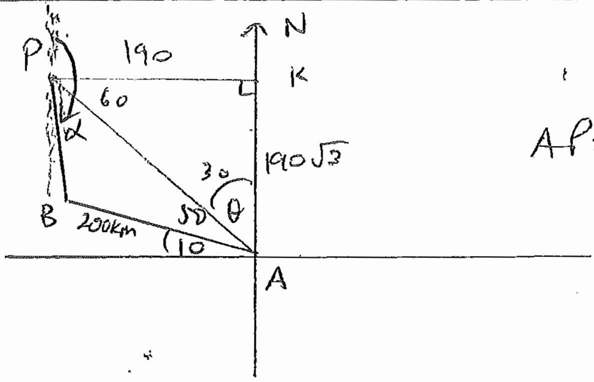
$t = \dfrac{\ln \frac{1}{50}}{-\frac{1}{5}\ln\left(\frac{5}{3}\right)}$ where $k = \frac{1}{5}\left(\ln\left(\frac{5}{3}\right)\right)$

$t = 38.29\ldots$

$\therefore$ 39 years for it to fall below \$1000

Academic Year	12	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

b(i)



$AP = \sqrt{190^2 + (190\sqrt{3})^2}$

$= 380$ km.

(ii) $\tan\theta = \dfrac{190}{190\sqrt{3}}$

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\theta = 30°$. $\therefore \angle KAP = 30°$.

(iii) $\angle PAB = 50°$

$PB^2 = 200^2 + 380^2 - 2 \times 200 \times 380 \times \cos 50$

$= 86696.283\ldots$

$PB = \sqrt{86696.283\ldots}$

$= 294.442326\ldots$

$\doteqdot 294$ km

(iv) $\angle APB$: $\dfrac{\sin\alpha}{200} = \dfrac{\sin 50}{294.442326\ldots}$

$\sin\alpha = \dfrac{200\sin 50}{294.442326\ldots}$

$\alpha \doteqdot 31°21'$

$\therefore$ Bearing is $180 + 31°21'$

$= 181°21'$.

Academic Year	12	Calendar Year	2015
Course	2unit Maths	Name of task/exam	Trial HSC

c. (i) $y = \log_e(2x-5)$

at $x=6$ $\quad y = \log_e(12-5)$

$y = \log_e 7 \qquad B(6, \ln 7)$

at $y=0$ $\quad 0 = \log_e(2x-5)$

$e^0 = 2x-5$

$1 = 2x-5$

$2x = 6$

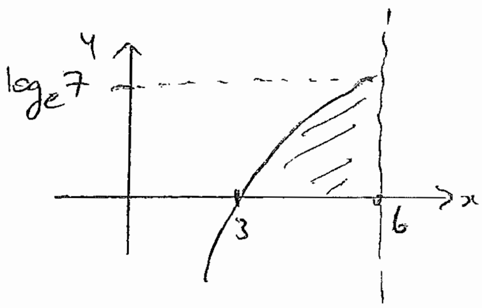
$x = 3 \qquad A(3,0)$

(ii) $y = \log_e(2x-5)$

$e^y = 2x-5$

$2x = e^y + 5$

$x = \dfrac{e^y+5}{2}$



(iii) $\text{Area}_{\text{Rect}} = 6 \times \log_e 7$

$= 6\log_e 7$

Area bound by y-axis:

$$A = \int_0^{\ln 7} \frac{e^y+5}{2}\,dy = \frac{1}{2}\Big[(e^y + 5y)\Big]_0^{\ln 7}$$

$$= \frac{1}{2}\left[e^{\ln 7} + 5\ln 7 - (e^0 + 5(0))\right]$$

$$= \frac{1}{2}\left[7 + 5\ln 7 - 1\right]$$

$$= \frac{1}{2}\{6 + 5\ln 7\}$$

$$= 3 + \frac{5}{2}\ln 7.$$

$\therefore$ Shaded area $= 6\ln 7 - 3 - \frac{5}{2}\ln 7$

$= \frac{7}{2}\ln 7 - 3.$